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ORIGINAL ARTICLE

# Biomechanics of the Cornea

Albert Daxer


## ABSTRACT

The purpose of this article is to provide an applicable and easy-to-use mathematical model of the biomechanics of the cornea. The new spherical dome model considers not only the heterogeneity of the tunica of the eye and distinguishes structurally between cornea, limbus and sclera. It also implements the structural anisotropy inside the corneal stroma caused by the corneas lamellar structure as well as the asphericity of the corneal shape.


**Keywords:** Cornea, Biomechanics, Corneal stress, Corneal model, LASIK, LASEK, PRK, SMILE, Keratectasia, Spherical dome model.

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## INTRODUCTION

The understanding of the biomechanics of the cornea is most important for the understanding of the behavior of the cornea during and after corneal and refractive surgery as well as for the understanding of ectatic corneal diseases. The complexity of the situation including macroscopic considerations as well as ultrastructural informations makes it difficult to find an applicable analytical approach for a detailed biomechanical discussion of the corneal tissue. Therefore, it is constumary to use numerical methods including finite element calculations to understand what goes on in this unique tissue under particular biomechanical conditions. Such approaches are laborious and applicable usually to a particular situation only.[1] A commonly used analytical approximations to explain the biomechanical behavior of the cornea is the elastic sphere model after Laplace,[2] which is shown in Figure 1 and represented by the formula

$$\sigma = p\frac{r}{2d} \text{ (Laplace)}$$

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In this formula, σ is the tension (stress) within the stroma, p is the intraocular pressure (IOP), r is the corneal radius and d is the thickness of the cornea.

The model after Laplace does not consider structural and biomechanical heterogeneities of the tunica of the eye, such as cornea, limbus and sklera. It also considers neither the biomechanical and structural anisotropy inside the tissue, such as the lamellar nature of the corneal stroma nor the aspherically designed corneal shape which is characterized by the gradually increasing corneal radius when tracing from the central to the peripheral cornea.

Here I shall propose a spherical dome model as a new analytical approach which considers (i) the biomechanical distinction between cornea, limbus and sklera, (ii) the structural anisotropy and lamellar nature of the corneal stroma, and (iii) the asphericity of the corneal shape in order to overcome the three major limitation of the elastic sphere model after Laplace.

### Ultrastructure and Biomechanical Framework of the Cornea

The cornea is a unique tissue with unique properties, such as transparency, optical and refractive function, biomechanical function and immunological privilege. Most of these properties are the result of its ultrastructural organisation. Ultrastructural informations about the corneal tissue can be obtained by means of two complementary methods: first, microscopic investigations and in particular electron-microscopic investigations and second beam scattering techniques and in particular small-angle X-ray scattering experiments.[3] Local information about the geometrical appearance of individual ultrastructural elements, such as, e.g. collagen fibrils and their relation to the local surrounding environment, such as, e.g. neighboring fibrils are best investigated by transmission electron microscopy (TEM).[4] The collective information about the structural elements in question over a specific volume of the tissue, such as structural arrangement of the collagen fibrils including their average dimensions and distances within the tissue are complementary to microscopy.[5] The combination of both the complementary approaches allow insights into the structure and function of the cornea and helps to explain the transparency[6] as well as the biomechanical properties[7,8] of the tissue.

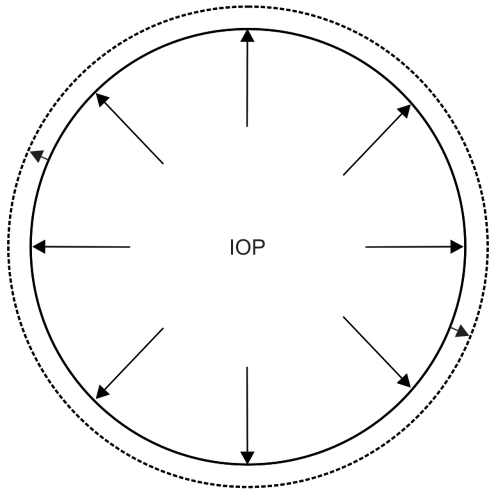



Fig. 1: Elastic sphere model after Laplace. The eye is approximated by a homogeneous, elastic sphere. The solid line represents the cornea and the dashed line indicates the elasticity of the tissue by marking the potential extended position of the cornea resulting from the load IOP

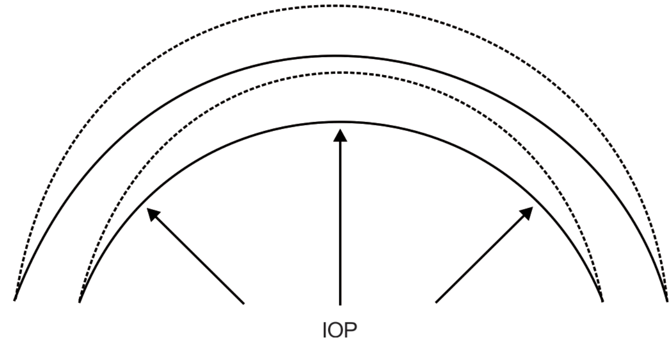



Fig. 2: Spherical dome model after Daxer. The limbus is represented as a nonelastic area to which the elastic cornea is fixed. The solid lines represent the cornea and the dashed lines indicate the potential extensibility of the tissue resulting from the load. There is no potential extensibility of the tissue at the limbus

About two-third of the collagen lammellae in a normal cornea are arranged orthogonally in a vertical-horizontal direction and one-third is randomly oriented.[5] This arrangement changes when approaching the corneal periphery at the limbus to a continous, circular arrangement of the collagen fibrils.[8,9] The regular, orthogonal collagen arrangement of the collagen fibrils in the normal cornea is destroyed in keratoconus.[7] This broken symmetry in the corneal meshwork is responsible for the reduced ability to biomechanically withstand the acting forces on the tissue. The heterogeneity of the collagen fibril organization at different parts of the wall of the eye, such as cornea, limbus and sklera is related to significant differences in the biomechanical properties among these different localizations. In particular, the change of the collagen fibril arrangement from the cornea to the limbus is responsible for the very high Young's modulus of 13 MPa at the limbus in comparison to only 0.3 MPa in the cornea and about 2 MPa in the sclera.[9-11] This information has dramatic consequences and limits the applicability of the mathematical model according to Laplace for the analysis of the biomechanical behavior of the cornea dramatically.

## The Spherical Dome Model

The amount of extensibility of a given tissue is related to the Young's modulus. Since, Young's modulus of the limbus is very much higher than the ones in cornea and sclera, the extensibility of the limbus as reaction to a given load can be considered as zero. The extensibility of the cornea $\Delta h/h$ in relation to the extensibility of the sclera $\Delta l/l$, behaves inversely to the related Young's moduli according to:

$$\frac{\Delta h}{\Delta l} = \beta \frac{Es}{Ec}$$

where Es is the Young's modulus of the sclera and Ec is the Young's modulus of the cornea and

$$\beta = \frac{h}{l}$$

According to the variation in the ultrastructural organization among different locations in the tunica of the eye as described above and the related differences in biomechanical properaties, such as Young's modulus between the central cornea and the limbus, the limbus can be considered as a border to which the cornea is fixed. In this model, the cornea is approximated by a dome (calotte of a sphere) with a radius of curvature r (representing the corneal radius) and a height h (representing roughly the anterior chamber depth) according to Figure 2.

In a first approach, I consider the resulting force vector on the cornea created by the load (IOP) which is directed virtually along the visual axis according to

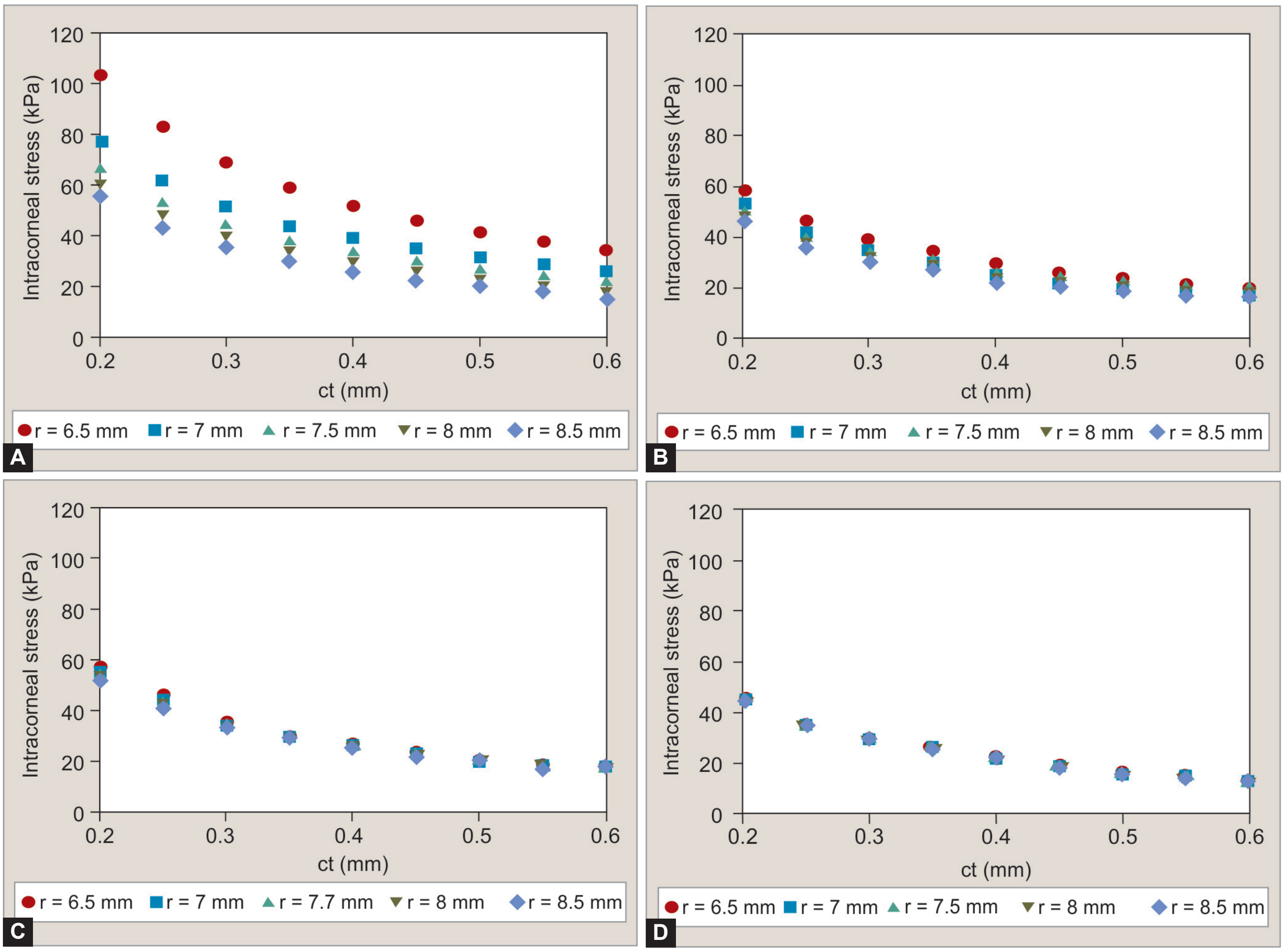
$$F1 = \pi p \frac{D^2}{4} \text{ (D is the diameter of the cornea)}$$

F1, which pushes the cornea outward, have to be compensated by a force F2 which attracts the cornea to the eye and protects the cornea from disruption. F2 represents the force according to the stress inside the tissue integrated over the cross-sectional area of the cornea according to

$$F2 = \pi D d \sigma$$

In fact, the effective cross-sectional area according to F2 which compensates F1 is the cross-sectional area multiplied by sin $(\alpha/2)$, where $\alpha$ represents the opening angle of the total cornea when approximating the cornea (dome) as a part of a sphere with

$$\sin \frac{\alpha}{2} = \frac{D}{2r}$$




**Figs 3A to D:** Intracorneal stress σ as a function of the corneal thickness for different central corneal radii according to the spherical dome model under different conditions: (A) Daxer 3 using $r_{eff}$ = r, (B) Daxer 4 using $r_{eff}$ = r, (C) Daxer 3 using $r_{eff}$ = (r + D)/2 and (D) Daxer 4 using $r_{eff}$ = (r + D)/2. IOP was set to 20 mm Hg and corneal diameter D to 12 mm

Therefore,

$$\sigma = \frac{pD}{4d}\left(1/\sin\left(\frac{\alpha}{2}\right)\right) = p\frac{r}{2d}$$

which is exactly the formula of the elastic sphere model after Laplace.[2]

The reason, why the approach of the spherical dome model after Daxer converges exactly to the elastic sphere model after Laplace is the fact, that no anisotropy within the tissue is considered. The force Vector F1 does not act on an isotropic corneal tisse but on a tissue which has to be described better by a direction dependent Young's modulus of the tissue (stress tensors) according to the corneal ultrastructure.[5] This anisotropic Young's modulus of the cornea is of course much higher in the lamellar direction (longitudinal strength) than in the transcorneal direction (cohesive strength). The force component of F1 in the lamellar direction is, therefore, much more 'effectively' compensated compared to the transcorneal direction. To implement the lamellar nature into the model, we have to make the following assumptions:

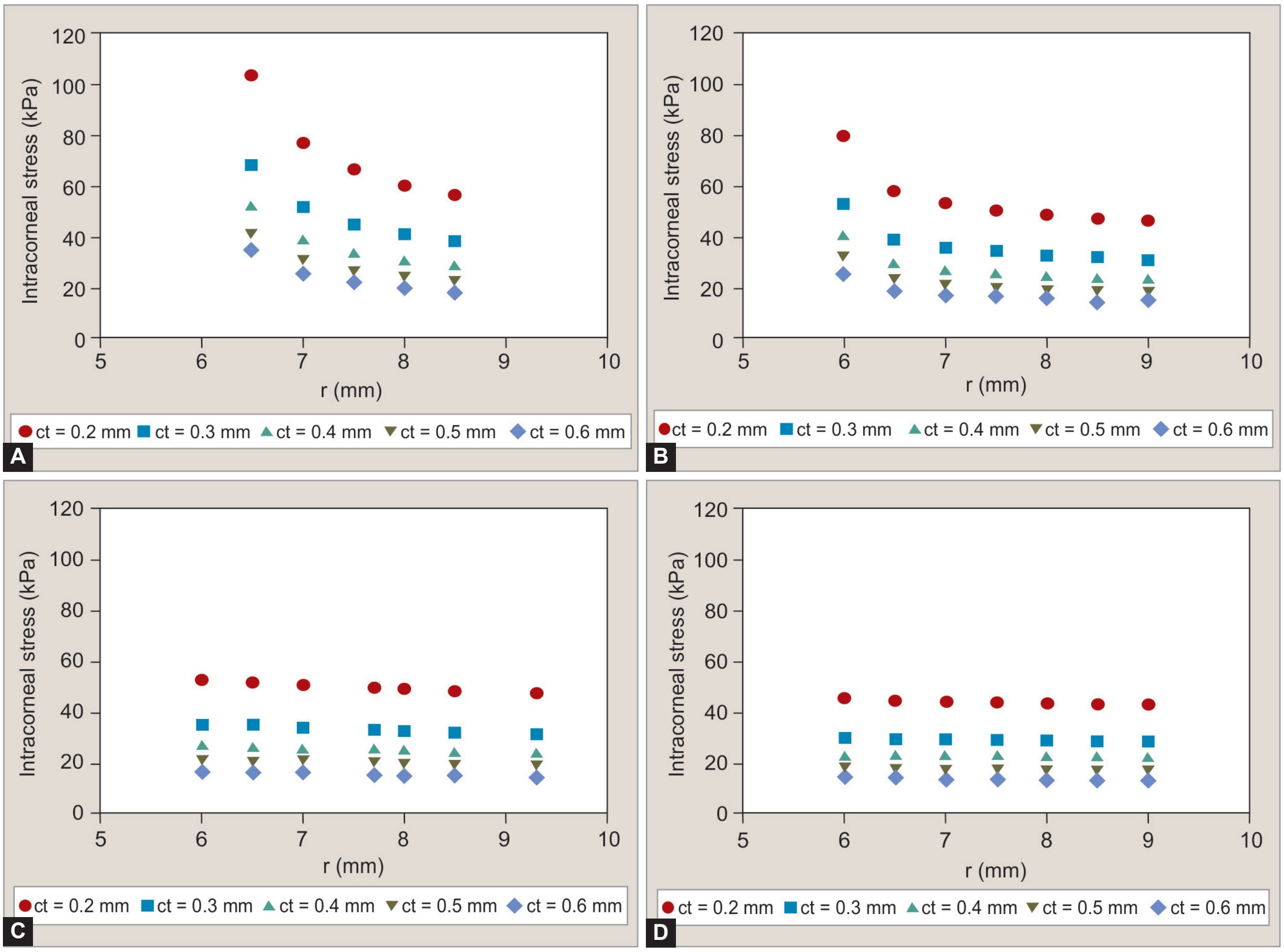
The direction of the force vector F1 deviates from the related projection of the cross-sectional area according to F2 in its effect on the intracorneal stress. In other words, the direction of the effectiveness of the load inside the tissue deflects somewhat from the direction of the load itself, according to the anisotropic lamellar nature of the corneal stroma. I approximate this fact by modifying the 'amount' of the corneal cross-sectional area contributing to the compensation of the load (F1) by replacing sin (α/2) by a anisotropy factor f which ranges between zero and 1 according to

$$\sigma = \frac{pD}{4d}\frac{1}{f} \text{ (Daxer 1)}$$

The easiest way to implement this anisotropy in the model is to remove the projection dependence of the cross-sectional area by removing sin (α/2) from the formula above and to privilege the lamellar direction over the transcorneal one, which results in

$$\sigma = \frac{pD}{4d} \text{ (Daxer 2)}$$

Here, the dependence on the radius (Laplace) changes to a dependence on the corneal diameter (Daxer 2). To




**Figs 4A to D:** Intracorneal stress as a function of the central corneal radius for different corneal thicknesses according to the spherical dome model under different conditions: (A) Daxer 3 using $r_{eff} = r$, (B) Daxer 4 using $r_{eff} = r$, (C) Daxer 3 using $r_{eff} = (r + D)/2$ and (D) Daxer 4 using $r_{eff} = (r + D)/2$. IOP was set to 20 mm Hg and corneal diameter D to 12 mm

increase (maximize) this anisotropy effect one can even replace $\sin(\alpha/2)$ by $\cos(\alpha/2)$, which seems an appropriate approximation especially if one considers the cornea as beeing just the 'top' of the spherical globe where $\cos(\alpha/2)$ is close to 1 according to the lamellar direction of the collagen fibrils being virtually orthogonal to F1 there. The resulting formula is:

$$\sigma = \frac{\frac{pD}{4d}}{\sqrt{1-\frac{D^2}{4r^2}}} \quad \text{(Daxer 3)}$$

Where

$$\cos\frac{\alpha}{2} = \sqrt{1-(D^2/4r^2)}$$

To implement the asphericity of the corneal shape in the model I equilibrate the corneal radii over the corneal surface by changing r into an effective corneal radius $r_{eff}$. The effective corneal radius $r_{eff}$ is somewhat between the central and the peripheral radius. Since, the peripheral corneal radius is not always clinically available a good approximation should be a value close to the diameter or axial length of the eye, which is also in the range of the corneal diameter D. The use of the corneal diameter D for this approximation has the advantage, that it already exists in the formulas and the introduction of a further variable is not required in order to keep the equations as simple as possible. A good approximation is, therefore, $r_{eff} = \frac{D+r}{2}$ instead of r in the formulas above and below.

In a second approach, the load-related force F1 acting at every point orthogonally to the inner corneal surface is compensated by the force F2 which results from the stress inside the cornea and the cross-sectional area without projection.

When considering the entire inner corneal surface area one gets the force F1 resulting from the IOP acting on the cornea according to

$$F1 = 2\pi pr^2\left[1-\sqrt{1-\frac{D^2}{4r^2}}\right]$$

and

$$F2 = \pi Dd\sigma$$

**Table 1:** Comparison of the relative postoperative weakness in percentage of the preoperative corneal strength for three different treatment modalities (SMILE, PRK, LASIK) for different corneal models

| | *Laplace* | *Reinstein, Archer and Randleman*[12] | *Daxer 3* $r_{eff} = r$ | *Daxer 3* $r_{eff} = (r + D)/2$ | *Daxer 4* $r_{eff} = r$ | *Daxer 4* $r_{eff} = (r + D)/2$ |
|---|---|---|---|---|---|---|
| SMILE | 68% | 75% | 99% | 85% | 89% | 83% |
| PRK | 68% | 68% | 99% | 85% | 89% | 83% |
| LASIK | 51% | 54% | 76% | 64% | 67% | 63% |

Which results in

$$\sigma = 2p\frac{r^2}{Dd}\left[1-\sqrt{1-\frac{D^2}{4r^2}}\right] \text{ (Daxer 4)}$$

when F1 = F2.

Here, the implementation of an effective corneal radius according to the explanations above can also be helpful for the consideration of the an aspheric corneal shape by replacing r by $r_{eff}$.

## RESULTS

Figures 3A to D show the dependence of intracorneal stress $\sigma$ as a function of the corneal thickness ct = d for different corneal radii for the model variant Daxer 3 and 4.

Figures 3A to D show, that the stress increases with decreasing corneal thickness. This effect depends quantitatively very little on the value of the pre-existing corneal curvature.

Figures 4A to D show the dependence of the intracorneal stress on the corneal curvature for different values of corneal thickness. It shows that the stress decreases slightly if the radius of the curvature increases. This effect is more prominent for smaller corneal thicknesses.

## DISCUSSION

Figures 3 and 4 show the dependence of the intracorneal stress of the spherical dome model after Daxer 3 and 4 on both, corneal curvature and corneal thickness. In contrast to the elastic sphere model after Laplace, which considers the eye as a homogenous elastic sphere without distinguishing between cornea, limbus and sclera (Fig. 1), the spherical dome model after Daxer implements, the biomechanical and ultrastructural heterogeneity of the tunica of the eye (i.e. distinguishing between corneal, limbus and sclera) as well as the anisotropy of the corneal ultrastructure (i.e. the lamellar nature of the cornea). By introducing the effective corneal radius $r_{eff}$ into the models, it should also be possible to consider at least partially the asphericty of the cornea in the calculations.

Applying the spherical dome model after Daxer as well as the elastic sphere model after Laplace for comparison to a case of SMILE, PRK and LASIK treatment of preoperatively –7.75 diopters and 550 microns corneal thickness as calculated by a very nice mathematical model after Reinstein, Archer and Randleman,[12] one gets the data shown in Table 1. It was assumed that the LASIK flap thickness was 110 microns without contributing to the corneal strength, the preoperative K-reading of 44 diopters and a tissue ablation of 100 microns for the –7.75 D correction.[13]

Reinstein et al consider in their model a depth dependent variation of cohesive strength and approximate the cohesive strength acting orthogonal to the lamellae beeing equal to the longitudinal tensile strength in the lamellar (fibrillar) direction. The model of Laplace as well as the spherical dome model after Daxer consider the mechanical properties as not being depth dependent and get, therefore, the same results for SMILE and PRK. In none of the models the LASIK flap contributes to the strength of the tissue. The elastic sphere model after Laplace as well as the spherical dome model after Daxer (except Daxer 2) consider a dependence of the intra-corneal stress from the corneal curvature. The model of Reinstein el al do not consider a dependence from the corneal curvature. It is interesting to note from Table 1, that Daxer 3 obviously underestimates the effect of corneal thinning on the corneal strength significantly when neglecting the asphericity of the cornea ($r_{eff} = r$). By considering the asphericity in Daxer 3 [($r_{eff} = (r + D)/2$)], the effect of corneal thinning on the corneal strength is predicted much more realistic. It seems, from Table 1, that the elastic sphere model after Laplace as well as the model after Reinstein et al may partially overestimate the effect of corneal thinning on the corneal strength relative to the spherical dome model after Daxer. One reason may be, that the spherical dome model after Daxer generally considers the anisotropy of the corneal tissue by the assumption that the Young's modulus and the strength in the lamellar direction (i.e. the direction along the collagen fibrils in the corneal stroma) is much higher than in the transcorneal direction (i.e. cohessive strength). This assumption is in agreement with the corneal ultrastructure.[5] The consequence is that the load (IOP) on the cornea is taken up by the lamellar oriented structures to a much higher extend compared to an isotropic consideration, which means that the model after Daxer predicts a 'stronger' cornea with less effect of corneal thinning on the postoperative corneal strength.

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